Full-Wave Analysis of a Lossy Rectangular Waveguide Containing Rough Inner Surfaces

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***Abstract*—The full-wave mode-matching method employing the similarity transformation to derive the eigenvalue equation for the periodic bumpy regions that model the surface roughness of a WR-10 (75–110 GHz) waveguide is presented. For the particular case study under the particular conditions applied, attenuation losses increase by approximately 60% (36%) at 75 GHz (110 GHz).**

I. Introduction

The presence of finite conductivity in a lossy rectangular waveguide results in a cross coupling between various TM and TE modes [1]. This implies that a hybrid mode formulation should be used for the rigorous field analysis of a lossy rectangular waveguide to obtain very accurate complex propagation constant. The existence of the rough or bumpy inner surfaces shown in Fig. 1, however, further complicates the problem. These bumpy inner surfaces introduce additional disturbance on the electromagnetic field distribution nearby and increase the propagation losses of the waveguide.

Morgan [2] reported a theoretic investigation on the effect of surface roughness of a semi-infinite conducting plate assuming periodic rectangular and triangular surface grooves. The power dissipation of the rough conductor plate was found to increase by about 60% over that for a smooth surface. Deventer, Katehi, and Cangellaris [3], on the other hand, applied the integral equation method using the equivalent surface impedance boundary condition to investigate the effect of surface roughness of a particular microstrip on the attenuation losses. Approximately 22% increase in attenuation constant was observed at 10 GHz.

In this letter, we will focus on the dominant mode propagation of a WR-10 rectangular waveguide as shown in Fig. 1. The surface roughness is modeled by the periodic rectangular grooves of period $r (r = p + q)$. The sidewalls, however, are assumed to be perfect conductors. Because we focus our attention on the effects of bumpy surface of finite conductivity σ on the dominant nearly TE_{10} mode, the simplified model is intended for physical understanding. The eigenvalue equation for the periodic bumpy region is derived by invoking a similarity transformation, which results in a simple close-form eigenvalue equation for determining the air modes and metal modes [4] in the bumpy surface region. Once these modes are obtained, the conventional network equivalent representation of the mode-matching method is applied to derive the final nonstandard eigenvalue equation [5], which results in the complex propagation constants.

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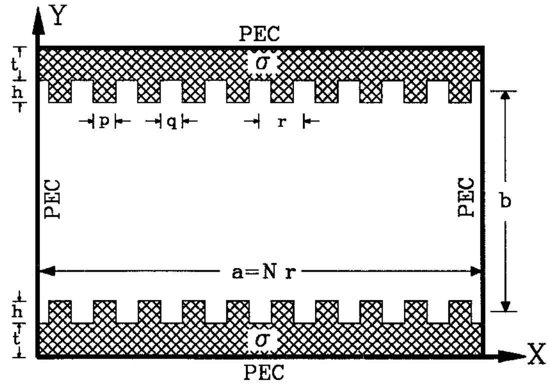

Fig. 1. Simplified model for a WR-10 rectangular waveguide with rough inner surfaces, $a = Nr = 2b = 2.54$ mm, $p = q = h$, $\sigma = 4.1 \times 10^7$ S/m and $t = 700$ μm. Each unit cell spans a distance r.

II. Dispersion Relation for the Bumpy Toothed Region

Since the mode-matching method is well known, only the derivation for the eigenvalue equation in the bumpy region, either $y \in (t, t+h)$ or $y \in (t+b, t+b+h)$, is briefly given. Fig. 2 shows the equivalent circuit representation of the bumpy region, which is the expanded view of the corresponding region in Fig. 1. Applying similar procedure described in [6], the electric and magnetic field vectors at both input and output interfaces of a unit cell are given by

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T] \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}, \tag{1}$$

where T is the known transfer matrix, and its determinant is equal to unity. Next, we apply the similarity transformation to diagonalize the matrix T. Thus, T becomes

$$[T] = [A][\Lambda][A]^{-1}, \tag{2}$$

where

$$[\Lambda] = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \tag{3a}$$

$$[A] = \begin{bmatrix} T_{12} & T_{12} \\ \lambda_1 - T_{11} & \lambda_2 - T_{11} \end{bmatrix}, \tag{3b}$$

and

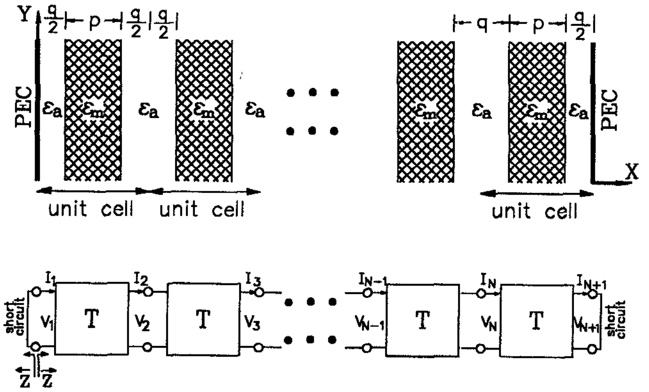

Fig. 2. Configuration of the bumpy region and its equivalent representation. Air region in each unit cell has a dielectric constant $\varepsilon_a(\varepsilon_a = 1)$ and a width $q/2$; Metal region in each unit cell has a width p and the complex dielectric constant σ_m charactrized by $1 - j\sigma/w\varepsilon_o$.

$$\lambda_{1,2} = \frac{(T_{11} + T_{22}) \pm \sqrt{(T_{11} + T_{22})^2 - 4}}{2}. \quad (3c)$$

Cascading N unit cells results in the N multiples of T. The resultant transfer matrix is

$$[T]^N = \begin{bmatrix} a_N & b_N \\ c_N & d_N \end{bmatrix} = [A] \begin{bmatrix} \lambda_1^N & 0 \\ 0 & \lambda_2^N \end{bmatrix} [A]^{-1}. \quad (4)$$

The resonance condition of this equivalent transmission line system requires that

$$\overleftarrow{Z} = -\overrightarrow{Z} = -\frac{b_N}{d_N} = 0. \quad (5)$$

Equation (5) is the eigenvalue equation for determining the corresponding air modes in the rectangular grooves and metal modes [4] in the teeth shown in Fig. 2, respectively.

III. Results

The convergence study, satisfying the relative convergence criterial [4], is investigated for a WR-10 waveguide with square grooves or teeth, i.e., $p = h = 127$ μm ($N = 10$ in Fig. 1). The calculation of phase constants and attenuation losses with accuracy better than 0.01% and 1.0%, respectively, requires that the number of metal modes in each toothed region is greater than 10, i.e., $n \geq 10$. Therefore, in the following analysis, $n = 10$ is used.

Fig. 3 plots the attenuation loss and the normalized phase constant of the nearly TE_{10} mode against frequency under various surface roughness conditions controlled by h/p ratio with $N = 20$ or $p = 63.5$ μm, where h is the depth of the rectangular groove. By varying the h/p ratio from 1.0 to 0.25 at -0.25 decremental step, the attenuation loss is consistently lowered and approaching the limiting case where $h = 0$, i.e., no bumpy surface is present, but smooth surface with finite conductivity σ. At 75 (110) GHz, for example, the attenuation loss increases by about 60% (36%) over that for the smooth surface.

Note that the increase of attenuation loss at $f = 75$ GHz is greater than that at $f = 110$ GHz when compared with those for the smooth surface. This can be explained by the fact that the magnetic fields H_x and H_z near the bumpy surface contribute to the attenuation loss of the near TE_{10} mode. In the case of a WR-10 waveguide with PEC inner surfaces,

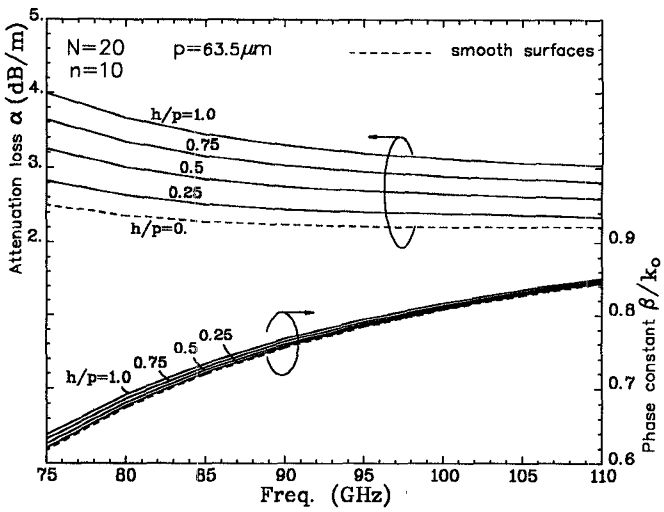

Fig. 3. Attenuation constant and the normalized phase constant of the nearly TE_{10} mode as a function of frequency under various surface roughness conditions controlled by h/p ratio with $N = 20$, or $p = 63.5$ μm.

$|H_x|^2 \propto (1 - f_c^2/f^2)$, where f_c is the cutoff frequency of the waveguide, and H_x is related to the longitudinal current flowing along the z-direction on the bumpy surface. On the other hand, $|H_z|^2 \propto f_c^2/f^2$ and H_z is related to the transverse current flowing along the bumpy surface. Therefore, the transverse (parallel) current, proportional to $H_z(H_x)$, increases (decreases) as frequency decreases. As Morgan [2] pointed out that the transverse grooves have a considerably greater adverse effect on attenuation loss than grooves parallel to the current, the combined effects of transverse and parallel surface currents will result in higher attenuation losses near the WR-10 waveguide cutoff frequency, approximate by 59.0 GHz.

IV. Conclusion

Rigorous investigation on the surface roughness effects of a WR-10 rectangular waveguide is presented. The new full-wave approach employs a similarity transformation to derive the eigenvalue equation for the periodic bumpy regions that model the surface roughness of the WR-10 waveguide. A particular case study of the surface roughness effects is reported and its results are discussed in detail.

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